

Systems I
Fall 2014
Sample MID-TERM EXAM

This is a closed-book, closed-note exam. Answer all of the questions **clearly, completely, and concisely**. You have 50 minutes, so be sure to use your time wisely. All work should be written in your blue book.

- (20 points) Prove that NOR is a *universal operator*. That is, prove that any logic function is equivalent to some expression that uses only the NOR operator. You may demonstrate equivalence of NOR with other operators using Boolean algebra or (labeled) circuit diagrams.
- (20 points) Consider the logic function described by the following truth table:

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

- Write this function in *disjunctive normal form*.
- Simplify this expression by using a *Karnaugh map*.

3. (30 points) Create a circuit that calculates $a < b$, where a and b are 4-bit, two's-complement values. This circuit should have a 1-bit output that is 1 when $a < b$, and 0 otherwise. **Hint:** Note that $a < 0 \leftrightarrow a - b < 0$, implying that subtraction is likely a useful operation here.

Extra challenge: If you use subtraction to solve this problem, then you must consider the behavior of your solution when the subtraction yields overflow. Can your circuit provide the correct answer for all values of a and b , even in the presence of overflow? If so, how? If not, why not?

4. (30 points) **Draw a circuit** that emits the following repeating 2-bit sequence:

00, 01, 00, 10, 00, 11 . . .

Your circuit should have 2 bits of output, as well as incoming CLOCK and CLEAR inputs. Every 6 cycles of the clock should progress through this sequence, which should “wrap around” and repeat with the next 6 clock cycles. You may use basic memory devices such as *flip-flops* (which may be aggregated into *registers*).